Economic growth, energy consumption and emissions: an extension of Ramsey-Cass-Koopmans model under EKC hypothesis[1]

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Abstract
This paper extends Ramsey-Cass-Koopmans growth model, including energy consumption and proxies for environmental quality. The analytic solution of the extension's maximization problem had shown through two methodologies (algebraic and by a computational simulation) that emissions are positively related with capital accumulation (as well as income). Corroborating the empirical studies found on the literature. It also has been defined the conditions for a EKC (Environmental Kuznets Curve) situation (named here as the green golden rule).
Key-words: economic growth, energy consumption, CO_2 emissions, Ramsey-Cass-Koopmans model.
JEL: Q56, O13

Resumo
O presente trabalho estende o modelo de crescimento econômico de Ramsey-Cass-Koopmans, incluindo o consumo de energia e uma proxy para qualidade ambiental. Usando duas diferentes metodologias – uma algébrica e outra através de simulação computacional – as soluções analíticas do problema de maximização proposta mostram que as emissões são positivamente correlacionadas com a acumulação de capital (o mesmo ocorre com a renda). Esses resultados corroboram as evidencias empíricas apresentadas na literatura. Foram também definidas as condições para a existência de uma EKC (Environmental Kuznets Curve) para a emissão de CO_2 (aqui chamada de condições para a regra de ouro verde).

Palavras-chave: crescimento econômico, consumo de energia, emissões de CO_2, modelo de Ramsey-Cass-Koopmans.
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Área 10 – Economia Agrícola e do Meio-ambiente

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1. INTRODUCTION

Energy is a fundamental resource in the economy. Any activity requires energy in some form. Consequently, economic growth is directly related to energy consumption and affected by its availability. On the other hand, the use of energy generates negative impacts. Externality caused by pollutants resulting from combustion processes (specially in case of fossil fuels), the possibility of exhaustion and, as a consequence, the risk of energy shortages in the future are just few examples.

The importance of the relationship between energy consumption, environmental quality and economic growth is reflected by its wide discussion within and outside Academics. The Environmental Kuznets Curve (EKC) is one special case among several interesting theoretical developments.

EKC is a model largely explored in recent years. Theoretical and empirical studies have shown that the relationship between income and the use of natural resources – or the environmental quality – may be described by an inverted U-shaped relationship (SHAFIK & BANDYOPHADYAY, 1992; GROSSMAN & KRUEGER, 1993; WORLD BANK, 1992, and SELDEN & SONG, 1994). According to stylized facts, the inverted "U-shaped" relationship results from interactions of several effects. The most important are: consumer demand for environmental quality, des-industrialization and development of new and more efficient technologies.

However, there are situations where EKC does not seem to occur. The energy consumption is one example in which EKC does not show perspective of a short-run turning-point (RICHMOND & KAUFFMAN, 2006).

Problems related to property rights, market failures, politics and polices not related to people's will, geographical and climate barriers and even forces out of human control seems to significantly affect the relationship between environmental quality and income (SHAFIK & BANYOPHADYAY, 1992; SHAFIK, 1994; SELDEN & SONG, 1994).

Besides these theoretical limitations, EKC has received many methodological criticisms. Unfortunately, many models are built on weak econometrics. Problems such as spurious correlation, incomplete models, omitted variables and lack of cointegration tests are commonly found on EKC studies (PERMAN & STERN, 2003; STERN, 2004). Models use static comparative approach, estimated on a reduced form and income is used as exogenous variable. The use of a structured dynamic model that incorporates the general equilibrium approach for the economy can generate a better, stronger and complete analysis.

One interesting model is the Ramsey-Cass-Koopmans growth model. It is a dynamic model, well explored and applied in several situations. It is a neoclassical growth model based upon the consumer's intergenerational utility maximization (BARRO & SALA-I-MARTIN, 2004; ROMER, 1996). Its usual application is the evaluation of macroeconomic polices but it is also useful to estimate the effect and interaction between macroeconomics with microeconomics issues (such as the environmental quality).

The main objective of this paper is to theoretically develop an extension for the neoclassical Ramsey-Cass-Koopmans dynamic model including the relationship between economic growth, energy consumption and environmental quality. As a secondary objective, it simulates a case situation using the model parameters traditionally found on the literature by the method of analytic solution.

2. THE MODEL

Traditionally, the Ramsey-Cass-Koopmans model problem is represented by the maximization of a (infinitely lived) intergenerational household's utility function represented by (eq. 1) (BARRO & SALA-I-MARTIN, 2004; ROMER, 1996):

$$\max U = \int_0^\infty u(t) \cdot \exp\left[-(\rho - n) \cdot t\right] dt \qquad \text{(eq. 1)}$$

Where:

$u(t)$ is the utility per person in the period of time t;
n is the rate of population growth; and,
ρ is the rate of intertemporal preference $(\rho > 0)$.

In this model, instant utility function has two components: the consumption per person and the environmental quality[1] (STOKEY, 1998). The utility function is assumed to be perfect separable between consumption and environmental quality. The utility function is represented by (eq. 2):

$$u_t(\hat{c}_t, X_t) = v(\hat{c}_t) - h(X_t) \qquad \text{(eq. 2)}$$

Where:

[1] In this study, this variable is represented by the net flow of CO_2.

$\hat{c}_t$ is the intensive form[2] per capita consumption in period t;
$X(t)$ is the net flow of CO_2 in period of time t;
v is increasing and strictly concave $\left(\lim_{c\to 0} v'(c)=\infty\right)$; and,
h is increasing and strictly convex $\left(\lim_{X\to 0} h'(X)=0\right)$.

Assuming Constant Intertemporal Elasticity of Substitution (CIES) on consumption and externality stock return to utility (BARRO & SALA-I-MARTIN, 2004), (eq. 2) is re-written as:

$$u_t(\hat{c}_t, X_t) = \frac{\hat{c}_t^{\,1-\sigma}-1}{1-\sigma} - \frac{B}{\gamma}(X_t)^{\gamma} \qquad \text{(eq. 3)}$$

Where:
$B>0$; $0<\sigma<1$ and $\gamma>1$.

Assuming that i) a Cobb-Douglas production function with a labor augmenting technology[3]; ii) split of physical capital in intensive and non-intensive on energy use (RASMUSSEN, 2001); and, iii) perfect substitutability and additive separable, the production function can be written as:

$$f\left(\hat{k}_t\right) = \left[\hat{ke}_t + \hat{kne}_t\right]^{\alpha} \qquad \text{(eq. 4)}$$

Where:
$\hat{k}$ is the intensive form (per capita) capital stock;
$\hat{ke}$ represents (per capita) stock of capital, intensive in energy;
$\hat{kne}$ is (per capita) stock of capital, non-intensive on energy; and,
$\partial\hat{kne}/\partial e=0$ and $\partial\hat{ke}/\partial e>0$ (e represents per capita energy consumption).

In each period t, the net flow of externality is assumed to be function of the flow of pollutant (F_t) minus a natural environmental recovery rate [4], as follows:

$$\dot{X}_t = (1-\eta)\cdot F_t \qquad \text{(eq. 5)}$$

STERN (2004) proposes that F_t be function of the product level (Y_t), the energy intensity (INT_t), the rate of pollutants generated by unit of energy consumed for each source of energy (tg_j), and the share of J sources on the energy matrix $\left(part_j = e_{jt}/E_t\right)$. In this

[2] $\hat{c}_t = c_t \cdot e^{xt}$.
[3] This is the only way a steady-situation is guaranteed (BARRO & SALA-I-MARTIN, 2004).

[4] This is similar to KELLY (2003).

study, however, the product $Y_t \cdot INT_t$ has been substituted by the capital stock intensive on energy use (KE_t). Thus, the flow of pollutants is represented by (eq. 6):

$$F_t = KE_t \cdot \sum_{j=1}^{J} \left(tg_j \cdot part_{jt} \right) \quad \text{(eq. 6)}$$

From (eq. 5) and (eq. 6), the rate of change in the stock of CO_2 is defined by:

$$\dot{X}_t = (1-\eta) \cdot KE_t \cdot \sum_{j=1}^{J} \left(tg_j \cdot part_{jt} \right) \quad \text{(eq. 7)}$$

The constraint on household's budget is the same as in the traditional Ramsey-Koopmans model. But, assuming that the physical capital is split into two types (eq. 4) and incorporating the competitive firms hypothesis, the flow of capital is given by:

$$\dot{\hat{k}}_t = \left[\hat{ke}_t + \hat{kne}_t \right]^{\alpha} - (x+n+\delta) \left[\hat{ke}_t + \hat{kne}_t \right] - \hat{c}_t \quad \text{(eq. 8)}$$

Where:

x is the rate of technological change $(x > 0)$; and,

δ is the rate of capital depreciation ($\delta > 0$).

The optimization problem is to choose the path for consumption, stock of capital and energy consumption[5] that maximizes the utility of the infinitely lived representative household. In other terms, the problem is to choose (c, e, k) that maximizes (eq. 9) and binds the transversality condition[6]:

$$\max \int_0^{+\infty} \exp\left[-(\rho - n) \cdot t \right] \cdot \left[\frac{\hat{c}_t^{\,1-\sigma} - 1}{1-\sigma} - \frac{B}{\gamma} (X_t)^{\gamma} \right] dt$$

$$s.t.$$

$$\dot{\hat{k}}_t = \left[\hat{ke}_t + \hat{kne}_t \right]^{\alpha} - (x+n+\delta) \left[\hat{ke}_t + \hat{kne}_t \right] - \hat{c}_t \quad \text{(eq. 9)}$$

$$\dot{X}_t = (1-\eta) \cdot KE_t \cdot \sum_{j=1}^{J} \left(tg_j \cdot part_{jt} \right)$$

$$\lim_{t \to \infty} \hat{k}_t \cdot \exp\left[-\bar{r}(T) \cdot T \right] = 0$$

The maximization problem is represented by the following Hamiltonian:

[5] Notice that it is not possible to choose the energy matrix, but just the energy intensity.

[6] This restriction implies that assets have non-negative present-values (BARRO & SALA-I-MARTIN, 2004).

$$H=\exp\left[-(\rho-n)\cdot t\right]\cdot\left[\frac{\hat{c}_t^{\,1-\sigma}-1}{1-\sigma}-\frac{B}{\gamma}(X_t)^{\gamma}\right]+$$
$$+\lambda_{1t}\left[\left[\hat{ke}_t+\hat{kne}_t\right]^{\alpha}-(x+n+\delta)\cdot\left[\hat{ke}_t+\hat{kne}_t\right]-\hat{c}_t\right]+ \qquad \text{(eq. 10)}$$
$$+\lambda_{2t}\left[(1-\eta)\cdot k\hat{e}_t\cdot\hat{L}_t\cdot\sum_{j=1}^{J}\left(tg_j\cdot part_{jt}\right)\right]$$

Where:

$\lambda_{1,2t}$ are shadow-prices of capital (1) and externality stock (2); and,

$B,x,n,\delta,\rho,\eta>0;\ 0<\sigma<1$ e $\gamma>1$.

The first-order equilibrium conditions, binding the transversality condition and assuming constant marginal 'des-utility' of externality stock on each period of time, is given by [7]:

$$\frac{\dot{\hat{c}}}{\hat{c}}=\left(\frac{1}{\sigma}\right)\cdot\left(\begin{array}{l}\alpha\cdot\hat{k}_t^{\alpha-1}-x-\delta-\rho+\\+\left(\lambda_{2t}\cdot(1-\eta)\cdot Z-B\cdot X_t^{\gamma-1}\right)\cdot\dfrac{\partial k\hat{e}_t}{\partial\hat{k}_t}\cdot\dfrac{1}{\lambda_{1t}}\end{array}\right) \qquad \text{(eq. 11)}$$

By assumption, in the early stages of economic growth, the marginal benefits of emissions are larger than their marginal costs (des-utility) (STOKEY, 1998). Thus, the use of energy-intensive form of capital grows in the beginning of the transition to a more developed economy. Which means that, in the limit, the change in capital will occur in the form of capital intensive in energy use. Which implies that: $0<\frac{\partial k\hat{e}}{\partial\hat{k}}<1$. The utility is assumed to be increasing and strictly concave in consumption $\left(\lim_{c\to 0}v'(c)=\infty\right)$ and increasing and strictly convex with respect to pollution $\left(\lim_{X\to 0}h'(X)=0\right)$. This implies that $\lim_{t\to\infty}\lambda_1=0$ and $\lim_{t\to\infty}\lambda_2=\infty$. As a consequence, along the capital accumulation path, consumption and emissions are monotonically increasing. Graphically, this translates as a steady-state consumption dynamics that moves to the right (Figure 1).

This expansion will be limited by two conditions: the golden rule of capital; or if faced by the Green Golden Rule - GGR (LE KAMA, 2001). The GGR is met whenever marginal benefit and cost of the emissions become equal[8]. Beyond this point it is not possible to extract an extra utility from emissions and further intensification on energy use does not occur. Under a Social Planner solution[9], this is a sufficient condition for an U-shaped EKC. From the GGR locus, reduction of CO_2 stock in the atmosphere (EKC for the stock) requires a marginal productivity of capital non-intensive in energy use larger than the sum of the rate

[7] See section 1 in the Appendix.

[8] The locus where $X_t=\left(\frac{1}{B}\cdot\hat{c}_t^{-\sigma}\cdot\frac{\partial\hat{c}_t}{\partial X_t}\right)^{\frac{1}{\gamma-1}}$.

[9] The underlying idea is a central decision maker that controls set of state variables (BARRO & SALA-I-MARTIN, 2004).

of technological change plus the rate of capital depreciation and the rate of intertemporal preference[10]: $\left(\alpha \cdot \left(\hat{kne}\right)^{\alpha-1}\right) > x + \delta + \rho$.

3 ANALYTICAL SOLUTION AND SIMULATION METHOD

The system of differential equations that describes the maximization problem (eq. 9) is represented by a model with two equations: optimum consumer behavior (eq. 11) and budget constraint (eq. 8). Two alternative methodologies were used to solve this system. The first is an algebraic analytical solution. The second is a graphical solution using a computational algorithm (TABARROK, 2000).

3.1 Algebraic analytical solution

Analytical solutions are the primitive functions of a dynamic system obtained using the integration calculus[11]. They are essential to observe the behavior of the economy and its dynamical variables along time (including the speed of convergence) and, also, to check the system's stability (BARRO & SALA-I-MARTIN, 2004).

The dynamic system that represents a Ramsey-Koopmans like problem is generally represented by (eq. 12):

$$\dot{y}(t) = A \cdot y(t) \qquad \text{(eq.12)}$$

Where:

$\dot{y}(t)$ is the vector that represents the dynamics of the state variables;

A is the matrix of constants; and,

$y(t)$ is the vector of state variables.

Assuming some boundary conditions, the analytical solution of system in (eq. 12) is represented by:

$$y(t) = -e^{-\varepsilon t} \qquad \text{(eq. 13)}$$

Where:

ε is the negative eigenvalues of matrix *A*.

Larger eigenvalues (in absolute terms) will imply faster convergence to the steady-state (BARRO & SALA-I-MARTIN, 1992). The essential result to guarantee the stability of the system is that the eigenvalues of matrix *A* have opposite signs.

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[10] This results from the GGR condition, assuming $\gamma > 1$ (Appendix, Section 2).

[11] For further discussion upon the analytical solution see CHIANG (1982) and BARRO & SALA-I-MARTIN (2004).

3.2 Computational algorithm

The solution for the Ramsey-Koopmans model that describes the behavior of the variables $\hat{k}$ and $\hat{c}$, were obtained using a set of algorithms in Mathematica® (Tabarrok, 2000) [12]. The coefficients α, x, n, δ, σ and ρ are from BARRO & SALA-I-MARTIN (2004) and ROMER (1996)[13]. The values of λ_2/λ_1 were controlled for two scenarios: 1 and 64. It has been included two starting points for $\hat{k}$ and $\hat{c}$: $\hat{k}(0)=1$ and $\hat{c}(0)=0.5$ and the saddle point path from a value close to the origin.

4 RESULTS

4.1 Algebraic analytical solution

The system of equations that describes the optimal behavior of the dynamics variables is:

$$\begin{bmatrix} d\left[\log\left(\hat{k}\right)\right]/dt \\ d\left[\log\left(\hat{c}\right)\right]/dt \end{bmatrix} = \begin{bmatrix} -\left(-(2-\alpha)\cdot\zeta + x + n + \delta\right) & \zeta - x - n - \delta \\ \dfrac{\alpha\cdot\zeta}{\sigma} & 0 \end{bmatrix} \cdot \begin{bmatrix} \log\left(\hat{k}/\hat{k}^*\right) \\ \log\left(\hat{c}/\hat{c}^*\right) \end{bmatrix} \quad \text{(eq. 14)}$$

Where:

$$\zeta = \left(\frac{1}{\alpha}\right)\left(\left(\lambda_{2t}\cdot(1-\eta)\cdot Z - B\cdot X_t^{\gamma-1}\right)\cdot\frac{\partial k\hat{e}_t}{\partial\hat{k}_t}\cdot\frac{1}{\lambda_{1t}} + x + n + \delta\right);$$

$\hat{k}^*$ and $\hat{c}^*$ are steady-state per capita stock of capital and consumption.

The eigenvalues that define the speed of convergence are obtained according to:

$$\varepsilon^2 + \left(-(2-\alpha)\cdot\zeta + x + n + \delta\right)\cdot\varepsilon - \frac{\alpha\cdot\zeta}{\sigma}\cdot\left[\zeta - x - n - \delta\right] = 0 \quad \text{(eq. 15)}$$

The value of coefficients α, x, n, δ, σ and ρ (used to extract the effect of the stock of pollution change on the speed of convergence) are also from BARRO & SALA-I-MARTIN (2004) and ROMER (1996). $\partial \hat{ke}/\partial\hat{k}$ are the variable controlled and λ_2/λ_1 was simulated on three scenarios (1, 4 and 16).

According to the simulation, the effect of a change in the stock pollution on the speed of convergence is increasingly in $\partial \hat{ke}/\partial\hat{k}$ and λ_2/λ_1. This relation is graphically presented in Figure 2. This means that the intensification of energy use, simulated by the

[12] The set of algorithms used are available by e-mail request.

[13] $\alpha = 1/3$; $x = 0.02$; n = 0.01; $\delta = 0.05$; $\sigma = 1.75$; and, $\rho = 0.02$.

increase of $\partial \hat{ke} / \partial \hat{k}$ and λ_2/λ_1, is positively related to convergence (larger levels of emission make the economy reach the steady-state faster). This implies that income growth, and its level, are positively related to emissions.

4.2 Result from computational model

According to (Figure 1) intensification of CO_2 emissions moves the steady-state of capital forward. This is confirmed by the results given by the computational algorithm (Figure 3 and 4). It also confirms suggestion on (Figure 2). According to (Figure 5), intensification of emissions raises the rate of economic growth and the speed of convergence.

5 CONCLUSIONS

The theoretical and analytical solutions (for both methods: algebraic and graphical) show that economic growth and flow of pollutants are positively correlated. Increasing flow of pollutants along economic growth implicates not only increasing stock of CO_2 in the atmosphere but also increasing speed of growth. This result confirms empirical studies that did not find an inverse-U-shaped EKC for energy and CO_2 emissions.

Thus, air pollution only will stop getting worse when the marginal benefits of CO_2 emissions equals its costs (the GGR – green golden rule). The determination of GGR locus, however, requires the definition of some parameters not found on traditional literature (specifically: B and γ). These parameters help define the GGR locus, but also, determine which comes first: GGR or the golden rule of capital. A suggestion for future studies is the estimation of their values.

It is interesting to notice that, although net emissions cease (from GGR) the stock of CO_2 in the atmosphere remains constant. Its reduction requires some other conditions. Those specific conditions could be a major problem if people's sensitiveness toward the costs of pollution is poorly underestimated. A problem because such result would overestimate the net benefit from pollution, pushing up the potential GGR locus, and until the stocks get stabilized or reduced to a lower level possibly the humanity would be under great negative influence of global warming. The present paper, however, doesn't want to discuss the negative effects of global warming, the focus is just algebraic derive the conditions and discuss some possible results.

As a final comment, the results of this study show that the market forces: i) does not optimally leads to a reduction on CO_2 emissions; ii) reduction of pollution stock in the atmosphere depends on people's behavior and some special conditions; and, iii) people's poor perception regarding emissions' cost can generate negative consequences in the future.

APPENDIX

Section 1. The first-order condition for equilibrium

Reallocating the terms in the Euler equation (eq. 10)[14], considering that $\hat{L}_t = e^{(x+n)\cdot t}$, $Z = \sum_{j=1}^{J} \left(tg_j \cdot part_{jt} \right)$ and $\partial part_{jt} / \partial k\hat{e} = 0$ results in:

[14] Taking the optimum levels of $\hat{k}$.

$$\frac{\dot{\lambda}_{1t}}{\lambda_{1t}} = -\left[\alpha \cdot \hat{k}_t^{\alpha-1} - (x+n+\delta)\right] + \begin{pmatrix} B \cdot X_t^{\gamma-1} \cdot \dfrac{\partial X_t}{\partial k\hat{e}_t} \cdot e^{xt} - \\ -\lambda_{2t} \cdot \left((1-\eta) \cdot e^{-\rho t} \cdot Z\right) \end{pmatrix} \cdot \frac{\partial k\hat{e}_t}{\partial \hat{k}_t} \cdot \frac{e^{nt}}{\lambda_{1t}} \tag{A.1}$$

Substituting (A.1) in the log operator of the first-order condition of (eq. 10) and assuming that the decision is always made in the present (t=0):

$$\frac{\dot{\hat{c}}}{\hat{c}} = \left(\frac{1}{\sigma}\right) \cdot \begin{pmatrix} \alpha \cdot \hat{k}_t^{\alpha-1} - x - \delta - \rho + \\ +\left(\lambda_{2t} \cdot (1-\eta) \cdot Z - B \cdot X_t^{\gamma-1}\right) \cdot \dfrac{\partial k\hat{e}_t}{\partial \hat{k}_t} \cdot \dfrac{1}{\lambda_{1t}} \end{pmatrix} \tag{A.2}$$

***Section* 2. Condition for reduction of pollution stock**

Assuming the utility function defined in (eq. 3), the equilibrium condition for marginal benefit and cost of the pollutants is given by:

$$\partial\left(\frac{\hat{c}_t^{1-\sigma} - 1}{1-\sigma}\right) \Big/ \partial X_t = \partial\left(\frac{B}{\gamma}\left(X_t\right)^{\gamma}\right) \Big/ \partial X_t \tag{A.3}$$

Or, equivalently:

$$\partial\left(\frac{\hat{c}_t^{1-\sigma} - 1}{1-\sigma}\right) \cdot \frac{1}{\partial \hat{c}_t} \frac{\partial \hat{c}_t}{\partial X_t} = B \cdot \left(X_t\right)^{\gamma-1} \tag{A.4}$$

Using the natural log, it becomes:

$$-\sigma \cdot \ln \hat{c}_t + \ln \frac{\partial \hat{c}_t}{\partial X_t} = \ln B + (\gamma - 1) \cdot \ln X_t \tag{A.5}$$

Taking the time differential results in:

$$-\sigma \cdot \frac{\dot{\hat{c}}}{\hat{c}} + \frac{\partial \dot{\hat{c}}}{\partial X} \Big/ \frac{\partial \hat{c}}{\partial X} = (\gamma - 1) \cdot \frac{\dot{X}}{X} \tag{A.6}$$

The relationship between per capita consumption and the stock of pollutants is constant in *GGR*, which means that $\frac{\partial \dot{\hat{c}}}{\partial X} \Big/ \frac{\partial \hat{c}}{\partial X} = 0$. Rearranging the terms gives:

$$\frac{\dot{X}}{X} = -\frac{\sigma}{(\gamma - 1)} \cdot \frac{\dot{\hat{c}}}{\hat{c}} \tag{A.7}$$

Substituting (A.2) in (A.7), given that $\partial k\hat{e} / \partial \hat{k} = 0$ and $\partial k\hat{e} / \partial t = 0$, the condition for $\dot{X} / X < 0$ is:

$$-\frac{1}{(\gamma-1)} \cdot \left[\alpha \cdot \left(\hat{kne} \right)^{\alpha-1} - x - \delta - \rho \right] < 0 \qquad \text{(A.8)}$$

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Figures

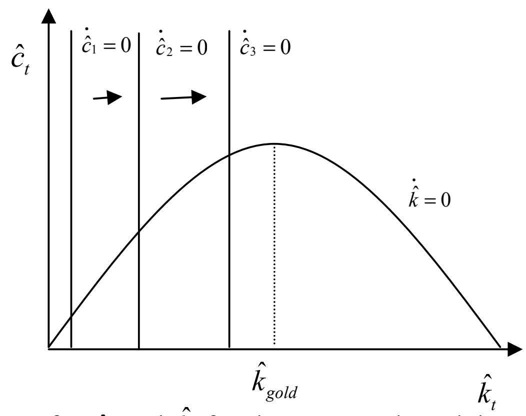

Figure 1 – Phase diagram for $\hat{c}$ and $\hat{k}$ for the proposed model.

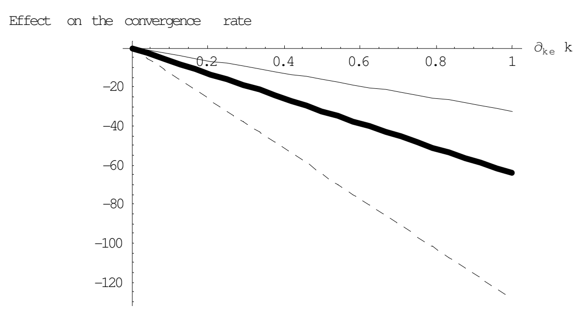

Figure 2 – Effect of the change on the level of stock of pollutants on the convergence rate.

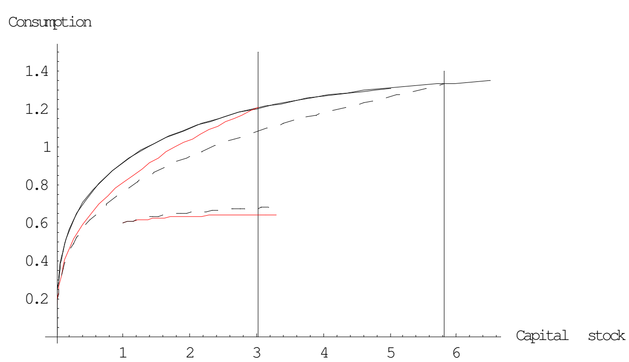

Figure 3 – Jointed phase diagram for the maximization problem (eq. 9) (the dashing dynamics represents the second scenario starting point).

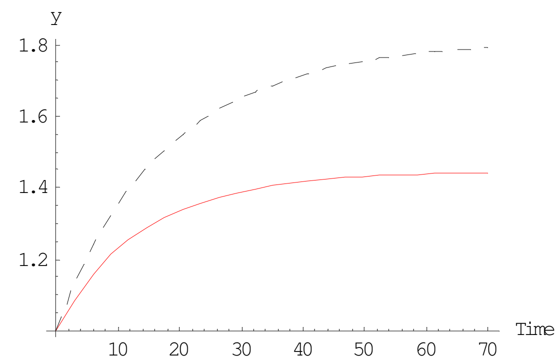

Figure 4 – Evolution of per capita income along time (the dashing dynamics represents the second scenario starting point).

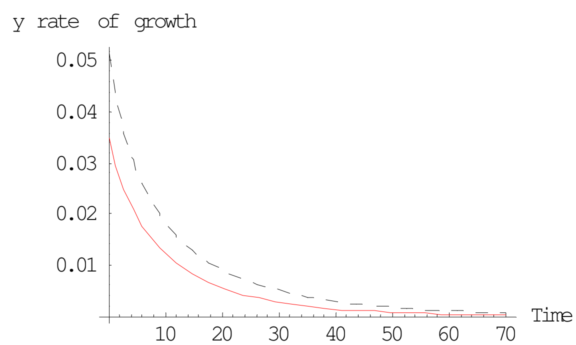

Figure 5 – Evolution of per capita income growth along time (the dashing dynamics represents the second scenario starting point).